To reduce an ODE of n^{th} order $(n>1)$ into a system of I^{st} order diff. equation.

Q). Reduce

$$3\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0 \quad \text{—— (A)}$$

into two first order differential equation.

Solution.

let $x = x_1$ —— (1)

and $\frac{dx}{dt} = x_2$ —— (2)

Putting value of x in (2)

$$\frac{dx_1}{dt} = x_2 \quad \text{—— (3)}$$

putting $\frac{dx_1}{dt} = x_2$ in (A)

$$3\frac{dx_2}{dt} + x_2 + x_1 = 0 \quad | \text{ from (1) \& (2)}$$

$$\frac{dx_2}{dt} = -\frac{1x_2}{3} - \frac{1}{3}x_1 \quad \text{—— (4)}$$

eq^n (3) and (4) are two first order ODE's which taken together is equivalent to given differential eq^n (A) of 2nd order.

Q). Show that an homogeneous ODE of order n with constant Co-efficients can be reduced to a system of n Ist order homogeneous ODE's.

Solution.

let $\frac{d^n x}{dt^n} + a_{n-1}\frac{d^{n-1}x}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}x}{dt^{n-2}}$

$+ \cdots\cdots + a_1\frac{dx}{dt} + a_0 x = 0$ —— (A)

be a given first order ODE

we assume $x_1 = x$ —— (1)

$$x_2 = \frac{dx}{dt} = \frac{dx_1}{dt} \quad \text{—— (2)}$$

$$x_3 = \frac{dx_2}{dt} = \frac{d^2x}{dt^2} \quad \text{—— (3)}$$

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$$x_n = \frac{d^{n-1}x}{dt^{n-1}} = \frac{dx_{n-1}}{dt} \quad \text{—— (n)}$$

putting the values of $\frac{dx}{dt}, \frac{d^2x}{dt^2}, \cdots \cdots \frac{d^{n-1}x}{dt^{n-1}}$ in (A), we get

$$\frac{dx_n}{dt} + a_{n-1}x_n + a_{n-2}x_{n-1} + \cdots + a_1 x_2 + a_0 x_1 = 0$$

$$\therefore \frac{dx_n}{dt} = -a_{n-1}x_n - a_{n-2}x_{n-1} - \cdots - a_1 x_2 - a_0 x_1 \quad \text{———} \quad (n+1)$$

Thus, the system of n first order homogeneous ODE given in (2) through $(n+1)$ is equivalent to the given n^{th} order homogeneous ODE (A)

Q). Convert the pair of 2nd order diff. equation

$$\frac{d^2y}{dt^2} + \frac{3dz}{dt} + 2y = 0 \quad \text{—— (A)}$$

$$\frac{d^2z}{dt^2} + \frac{3dy}{dt} + 2z = 0 \quad \text{—— (A)}'$$

Sol^n

let $y = x_1$ —— (1)

$$\frac{dy}{dt} = x_2 \quad \text{—— (2)}$$

$$z = x_3 \quad \text{—— (3)}$$

$$\frac{dz}{dt} = x_4 \quad \text{—— (4)}$$

Putting the value of $y, \frac{dy}{dt}, z, \frac{dz}{dt}$ from (1), (2), (3), (4) in (A) and (A)' we get

$$\frac{d(x_2)}{dt} + 3x_4 + 2x_1 = 0 \quad \text{—— (5)}$$

$$\frac{d}{dt}(x_4) + 3x_2 + 2x_3 = 0 \quad \text{—— (6)}$$

$\therefore$ eq^{ns} (2), (4), (5), (6) comprise a system of four homogeneous ODE's of first order which is equivalent to two second order homogeneous ODE's of ~~secon~~ given in (A) and (A)'